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REWARD STRUCTURE MODEL FOR FACULTY MEMBERS OF INDIAN UNIVERSITY

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ABSTRACT

Study of occupational mobility as an index of changes in society or an organization is of great importance for the proper planning of human resources. It's another aspect is the study of reward structure as it is fact that persons change their jobs primarily to earn a better income or to enjoy better status. Individuals get a reward for their existing post. Promotion is needed to get a better reward. In some situations the payment of reward stretches over a long time period, will happen discount in future income. Then reward structure with discounting may be considered. Otherwise reward structure without discounting is considered. The present work is an attempt to develop a model regarding the promotion and the corresponding rewards gained by the promotion considering both the situation with discounting and without discounting using the semi-Markov model.

KEYWORDS: *Expected Reward, Bonus, Holding Time, Discounting in Reward AMS Subject Classification: 60K15*

INTRODUCTION

Study of occupational mobility as an index of changes in society or an organization is of great importance for the proper planning of human resources. It refers to the transition of an individual or group or occupational vacancy, through the stratification system of societies or organizations. Most studies assume gradation or scaling in terms of occupational status or prestige against which the transition happens. Mobility through occupation primarily happens to improve social as well as occupational status. Another aspect is the study of reward structure as it is fact that persons change their jobs primarily to earn a better income or to enjoy better status. Individuals get a reward for their existing post. Promotion is needed to get a better reward. A same organization having the same set-up may use the same model to study the promotion pattern. The present work is an attempt to develop a model regarding the rewards gained by the promotion using the semi-Markov model.

Renumerate employees for their efficiency has been the keystone of industrial and business development for centuries. A financial reward has always been important in managing employee's accomplishment, but last few years other elements of reimbursement have developed to dispense employers with more scope to reward and therefore stimulate employees. Armstrong and Taylor (2014) [[1]] state that "Performance is defined as behaviour that accomplishes results. Performance management influences performance by helping people to understand what good performance means and by providing the information needed to improve it. Reward management influences performance by recognising and rewarding good performance and by providing incentives to improve it."

An individual or a group of employees who have been for a particular time period have had a particular job category or a post can get a reward. We generally consider reward as a random variable as it is associated with time, time is a random variable, thus reward is also considered as a random variable associated with the existing posts and promotions, but it will simply a lump sum money that an individual will get due to promotion. It will allow us to compute the functional features of the systems.

Individuals occupy a post for a particular time period, they will get a reward and they will get a bonus, a fixed sum when they change their posts at sometime. In some situations the payment of reward stretches over a long time period, will happen discount in future income. Then reward structure with discounting may be considered Otherwise reward structure without discounting is considered.

Many works was done on the reward structure using the semi-Markov process. Sladky(2004,2005, 2012) [[33, 34, 35]], Carravetta et al(1981) [[3]], De Dominicis et al(1984, 1986, 1991) [[12, 13, 14]] have worked on reward structure based on semi-Markov process. Benito F (1982)[[2]] has calculated the variance in the Markov process. Filar et al(1989) [[15]], Hueng and Kallenberg (1994)[[21]], Jaquette(1972,1973,1975)[[22, 23, 24]], Kadota(1997) [[26]] have developed series of paper on this same topic. Gosavi A Abhijit (2013) [[16]] has worked on the variance-penalized Markov decision process. Chattopadhyay(1989)[[5]] has worked on the reward model considering various job categories and has found occupational mobility measure based on the reward model.

In this present work, models have been developed on the basis of the promotion pattern of Indian University and it's the corresponding reward. Here semi-Markov continuous time reward model with discounting and without discounting are used to explain the whole process.

Model

We consider that there are N categories of posts of faculty members. Individuals will enter in the ith post at time τ (calendar time). They will enter in post i and will be promoted to post j according to the probability p_{ij}. They will stay in post i before getting the promotion to post j with a random time interval. Here the holding time density is $h_{ij}(\tau)$ at time τ. Individuals occupying post i having chosen the successor post j will earn a reward at a rate $y_{ij}(\sigma)$, at time σ (σ is a calendar year, σ 6= τ). $y_{ij}(\sigma)$ is the amount of money paid continuously for the occupancy of post i and is called a yield rate. When promotion occurs from i to j actually at some time τ, they earn bonus $b_{ij}(\tau)$, a fixed sum, is a lump sum payment at time of promotion for one time only, depending on the promotion and holding time in post i preceding the promotion.

We often have a situation where the payment of reward elongated over a long period, perhaps several years. It results in discounting in future income. A sum of money that will be received or paid in upcoming time costs less in current time because a smaller amount placed at interest beginning today could generate that larger sum in the future. We assume that a unit sum of money at time t in the future has a worth or present value of $\exp^{-\alpha\sigma}$ today, $\alpha \geq 0$. That is we shall carry out continuous discounting at the rate α.

It is to be noted that if individuals enter post i at time 0 and leave post i at time τ for entering the successor post j, then the value of the reward at time 0 generated by the yield rate structure will be

$$\int_0^\tau \exp^{-\alpha\sigma} y_{ij}(\sigma)d\sigma \qquad (1)$$

We may also represent the equation(1) as $y_{ij}(\tau,\alpha)$ where $y_{ij}(\tau,\alpha)$ is the discounted yield rate of an individual who is occupying post i having chosen the successor post j at holding time τ with discounting rate α. Thus,

$$y_{ij}(\tau,\alpha) = \int_0^{\tau} \exp^{-\alpha\sigma} y_{ij}(\sigma) d\sigma \qquad (2)$$

The exponential term is used to represent the discounted yield contribution to reward and it is considered for the calendar year 0 to τ, thus the product term is integrated for the interval 0 to τ to get the discounted reward for this interval.

The first promotion from post i can occur before or after the time t (length of service). If it occurs after t then the individuals will spend the entire interval of length t in post i. In this case they will gain rewards according to the yield rate structure for post i, throughout the time t and will gain terminal reward $v_i(0)$. So $v_i(0)$ be the terminal reward paid in the ith post at time 0 without discounting. The rewards from the yield structure are discounted with the upper limit of integral equal to t, the terminal reward $v_i(0)$ has a present value $\exp^{-\alpha t} v_i(0)$. The sums of these discounted rewards multiplied by the probability of their occurrence over all possible successor posts and over all holding times greater than t are taken. If on the other hand the first promotion occurs from post i to post j at time $\tau < t$, then the yield rate structure will give discounted rewards according to equation[1]. In addition, the bonus $b_{ij}(\tau)$ will be paid at time τ and will have a present value at the beginning of the interval of $\exp^{-\alpha\tau} b_{ij}(\tau)$. Finally, individuals will be promoted to post j when an amount $(t-\tau)$ of the original interval remains. The expected present value of future rewards at the time of entrance is therefore $v_j(t-\tau,\alpha)$. So $v_j(t-\tau,\alpha)$ be the future reward that will be paid in the successor post j at time $t-\tau$ with discounting rate α. When we observe this quantity at the beginning of length τ, it must be multiplied by the discount force $\exp^{-\alpha t}$ to account for the additional delay of τ in receiving these rewards. Summation of these reward terms multiplied by their probability of occurrence over all values of j and first transition times $\tau < t$ is then done. Both of these sums are again summed over the preceding post to get the expected present value of reward. Thus the expected present value of the reward gained by individuals will be [According to Howard (1971)] [[20]]

$$\begin{aligned} v_i(t,\alpha) &= \sum_{j=1}^{N} p_{ij} \int_0^{t} h_{ij}(\tau)[y_{ij}(\sigma) + \exp^{-\alpha t} v_i(0)] d\tau \qquad (3) \\ &+ \sum_{j=1}^{N} p_{ij} \int_0^{t} h_{ij}(\tau)[y_{ij}(\sigma) + \exp^{-\alpha\tau} b_{ij}(\tau) \\ &\qquad + \exp^{-\alpha\tau} v_j(t-\tau,\alpha)] d\tau \end{aligned}$$

Applying equation no (2), the expected present value of the reward for time t (length of service) will be

$$v_i(t,\alpha) = \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)[y_{ij}(t,\alpha) + \exp^{-\alpha t} v_i(0)]d\tau \tag{4}$$

$$+ \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)[y_{ij}(t,\alpha) + \exp^{-\alpha\tau} b_{ij}(\tau)$$

$$+ \exp^{-\alpha\tau} v_j(t-\tau,\alpha)]d\tau$$

In this present situation we assume that yield rate [$y_{ij}(\sigma)$], holding time [$h_{ij}(\tau)$], bonus rate [$b_{ij}(\tau)$] follows Exponential or Weibull distribution as time and money both follows in general Exponential or Weibull distribution, terminal reward is given and the future reward term is ignored due to the continuous change of Government policies. We consider the expected present value of reward at time t with discounting rate α in the ith post will be $R_i(t,\alpha)$. In this situation,

$$R_i(t,\alpha) = \sum_{j=1}^{N} p_{ij} \int_t^{t_0} h_{ij}(\tau)[y_{ij}(t,\alpha) + \exp^{-\alpha t} v_i(0)]d\tau \tag{5}$$

$$+ \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)[y_{ij}(t,\alpha) + \exp^{-\alpha\tau} b_{ij}(\tau)]d\tau$$

$$= \sum_{j=1}^{N} p_{ij} y_{ij}(t,\alpha) \int_t^{t_0} h_{ij}(\tau)d\tau + \exp^{-\alpha t} v_i(0) \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)d\tau$$

$$+ \sum_{j=1}^{N} p_{ij} \int_0^t [y_{ij}(t,\alpha) + \exp^{-\alpha\tau} b_{ij}(\tau)] h_{ij}(\tau)d\tau$$

Simulation

Here we have simulated the matrices p_{ij}, λ_{ij}, β_{ij} and γ_{ij}. We have assumed that yield rate [$y_{ij}(\sigma)$], holding time [$h_{ij}(\tau)$], bonus rate [$b_{ij}(\tau)$] follows Exponential or Weibull distribution.

Case: 1(a)

Here we assume that holding time [$h_{ij}(\tau)$] follows Exponential distribution with parameter λ_{ij} (Mukherjee and Chattopadhyay (1989)) [[31]], and Yield rate [$y_{ij}(\sigma)$], bonus rate [$b_{ij}(\tau)$] also follow Exponential distribution with parameters β_{ij}, γ_{ij} respectively. Initially we assume that holding time [$h_{ij}(\tau)$], yield rate[$y_{ij}(\sigma)$], bonus rate [$b_{ij}(\tau)$] are independent of i,j. Thus, $\sum_{j=1}^{N} p_{ij} = 1$ and holding time, yield rate, bonus rate parameters are λ, β and γ respectively. Therefore, the holding time distribution function is $h_{ij}(\tau) = h(\tau) = \lambda \exp(-\lambda\tau)$, the yield rate distribution function is $y_{ij}(\sigma) = y(\sigma) = \beta \exp(-\beta\sigma)$ and the bonus rate distribution function is $b_{ij}(\sigma) = b(\sigma) = \gamma \exp(-\gamma\sigma)$. We calculate expected a reward for different values of time t, terminal reward $v_i(0)$, holding time rate (λ), yield rate (β), bonus rate (γ). $R_i(t,\alpha)$, the expected reward gained by individuals in a time interval t if they are promoted to post i, at the beginning of the interval and the first promotion out of post i, can occur either before or after t. (Table:1) shows that with the increase of t, $R_i(t,\alpha)$ decreases.

Individuals will be in post i for the entire interval of length t if the promotion will take place after t. Then individuals will be promoted to the next post and benefited more. A smaller value of time interval to promotion from one post to another maximises expected reward. In this situation they will get a terminal reward. The reward will bemaximum with the increment of terminal reward. The result found in (Table: 2) clearly prove that fact.

An individual will be promoted to the next post when promotion occurs before t. If mean holding time in a particular post is reduced, individuals will get the maximum benefit. Table 3, shows that greater the value of λ, (i.e lower value of $\frac{1}{\lambda}$) results in a higher value of the expected reward. When the rate of yield increases, or in other words if the mean of yield rate decreases for a time t, then the expected reward will be increased for that time interval. But the increment rate will be very low. Yield rate does not affect much on an expected reward.(Table:4) has stated it clearly.

(Table 5) shows that if the bonus rate will increase simultaneously for a time interval, then the expected reward will increase simultaneously for that time interval, but in a very low amount.

Table 1: Expected Reward for Different Year When Holding Time Rate Follows Exponential Distribution

Discounting Rate (α)	Holding Time rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	2	3	4	100	05	20.2192
0.5	2	3	4	100	10	05.1497
0.5	2	3	4	100	15	03.9128
0.5	2	3	4	100	20	03.8022

Table 2: Expected Reward for Different Terminal Reward when Holding Time Rate Follows Expotential Distribution

Discounting Rate (α)	Holding time Rate (λ)	Yield rate (β)	Bonus rate (γ)	Terminal Reward (A)	time t	Expected Reward ($R(t,\alpha)$)
0.5	2	3	4	100	05	20.2192
0.5	2	3	4	150	05	28.4277
0.5	2	3	4	50	05	12.0107
0.5	2	3	4	05	05	04.6230

Table 3: Expected Reward for Different Holding Time Rate that Follows Exponential Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	2	3	4	100	05	20.2192
0.5	5	3	4	100	05	48.2906
0.5	10	3	4	100	05	94.2722
0.5	15	3	4	100	05	139.919

Table 4: Expected Reward for Different Yield Rate (Exponential) when Holding Time Rate Follows Exponential Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	2	3	4	100	05	20.2192
0.5	2	6	4	100	05	20.4170
0.5	2	10	4	100	05	20.5049
0.5	2	20	4	100	05	20.5746

Table 5: Expected Reward for Different Bonus Rate (Exponential) when Holding Time Rate Follows Exponential Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	2	3	04	100	05	20.2192
0.5	2	3	10	100	05	20.5884
0.5	2	3	15	100	05	20.7027
0.5	2	3	20	100	05	20.7662

Case: 1(b)

Here we assume that holding time [$h_{ij}(\tau)$], follows Weibull distribution with parameters λ_{ij} and k (widely used as time distribution), and yield rate [$y_{ij}(\sigma)$], bonus rate [$b_{ij}(\tau)$] follows Exponential distribution with parameters β_{ij} and γ_{ij} respectively. Initially we assume that holding time [$h_{ij}(\tau)$], yield rate[$y_{ij}(\sigma)$], bonus rate [$b_{ij}(\tau)$] are independent of i,j. Thus, $\sum_{j=1}^{N} p_{ij}=1$. Holding time that follows Weibull distribution with parameters λ and k, and yield rate, bonus rate follows Exponential distribution with parameters β, γ respectively. The holding time distribution function is $h_{ij}(\tau) = h(\tau) = \frac{k}{\lambda}(\frac{\tau}{\lambda})^{k-1}\exp(-\frac{\tau}{\lambda})^{k}$, the yield rate distribution function is $y_{ij}(\sigma) = y(\sigma) = \beta \exp(-\beta\sigma)$ and the bonus rate distribution function is $b_{ij}(\sigma)=b(\sigma) = \gamma \exp(-\gamma\sigma)$ and we calculate expected reward for different values of time t, terminal reward $v_i(0)$, rate of holding time (λ), rate of yield (β), rate of bonus (γ) with fixed discounting rate α and shape parameter k.

The expected reward $R_i(t,\alpha)$ in a time interval t gained by the individuals if they are promoted to post i at the beginning of the interval when first promotion out of post i can occur, either before or after t. Table:6 shows that, with the increase of t, $R_i(t,\alpha)$ decreases and after a long interval it will become constant

Individuals will be in post i for the entire interval of length t if the promotion will take place after t. They will be promoted to the next post if promotion occurs before t and they will benefit more. An expected reward will be maximised if time interval to promotion from one post to another is minimised. After certain years reward will be constant with the increase of time.

They will get a terminal reward when promotion takes place after t and individuals will be in post i for entire time interval t. Individuals will get the maximum reward with the increment of terminal reward. Table: 7 clearly prove that fact.

Individuals will be promoted to the next post when a promotion occurs before t. Individuals will get maximum benefit if the mean holding time in each post is decreased. Here we assume that holding time follows two parameters Weibull distribution with shape parameter k and scale parameter λ. If we consider shape parameter $k = 1$, it will lead to Exponential distribution with parameter λ. Table 8 shows that greater the value of λ, (i.e lower value of $\frac{1}{\lambda}$) results in a higher value of the expected reward. An expected reward is highly affected due to the holding time of individuals.

An expected reward has also affected due to the yield rate. If yield rate increases, or in other words if the mean yield rate decreases for a time interval, then the expected reward will increase for that time interval. In this case we can see that the increment rate is very low. That is yield rate does not affect much on the expected reward. Table: 9 clearly state this.

The expected reward will increase for a time interval, but in a very low amount if the bonus rate will increase for that time interval. Table: 10 show it clearly.

So from this overall study it is clear that if holding time in an existing post of individuals is minimised for the promotion to the next post, it will give the maximum reward and individual will be benefited more. So it is necessary for all individuals to know the actual promotion pattern of an institution so that they will get maximum reward.

Table 6: Expected Reward for Different Year when Holding Time Rate Follows Weibull Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	0.5	3	4	100	05	.944930
0.5	0.5	3	4	100	10	.874015
0.5	0.5	3	4	100	15	.874015
0.5	0.5	3	4	100	20	.874015

Table 7: Expected Reward for Different Terminal Reward when Holding Time Rate Follows Weibull Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	0.5	3	4	100	05	101.052
0.5	0.5	3	4	150	05	101.060
0.5	0.5	3	4	050	05	101.030
0.5	0.5	3	4	005	05	100.000

Table 8: Expected Reward for Different Holding Time Rate that Follows Weibull Distribution)

Discounting Rate (α)	Holding Time rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	0.3	3	4	100	05	60.8088
0.5	0.5	3	4	100	05	101.052
0.5	0.7	3	4	100	05	141.294
0.5	0.9	3	4	100	05	181.537

Table 9: Expected Reward for Different Yield Rate (Exponential) when Holding Time Rate Follows Weibull Distribution

Discounting Rate (α)	Holding Time Rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	0.5	3	4	100	05	101.052
0.5	0.5	6	4	100	05	201.659
0.5	0.5	10	4	100	05	335.802
0.5	0.5	20	4	100	05	671.159

Table 10: Expected Reward for Different Bonus Rate (Exponential) when Holding Time Rate Follows Weibull Distribution

Discounting Rate (α)	Holding Time rate (λ)	Yield Rate (β)	Bonus Rate (γ)	Terminal Reward (A)	Time t	Expected Reward ($R(t,\alpha)$)
0.5	0.3	3	04	100	05	101.052
0.5	0.5	3	10	100	05	101.718
0.5	0.7	3	15	100	05	141.274
0.5	0.9	3	20	100	05	181.829

An Example

Here holding time, yield and bonus are not independent of i,j. Holding time, yield and bonus are estimated from the data that have collected on the occupational data of full-time teachers of Midnapore College (Autonomous), West Bengal, India. We have considered year span 2000-2015. p_{ij} is also estimated from the above data. We get the following estimated matrices.

$$\widehat{P} = ((\widehat{p_{ij}})) = \begin{pmatrix} 0.0001 & 0.3667 & 0.2000 & 0.4333 \\ 0.0001 & 0.0001 & 0.5333 & 0.4667 \\ 0.0001 & 0.0001 & 0.1667 & 0.8333 \\ 0.0001 & 0.0001 & 0.0001 & 1.0000 \end{pmatrix}$$

$$\widehat{H} = \left(\left(\widehat{h_{ij}}(\tau)\right)\right) = \begin{pmatrix} 0.0001 & 5.1667 & 10.6300 & 13.5400 \\ 0.0001 & 0.0001 & 5.0500 & 7.6400 \\ 0.0001 & 0.0001 & 0.0001 & 2.9200 \\ 0.0001 & 0.0001 & 0.0001 & 1.0000 \end{pmatrix}$$

$$\widehat{Y} = ((\widehat{y_{ij}}(\tau))) = \begin{pmatrix} 1.000 & 2.000 & 3.000 & 4.000 \\ 1.000 & 3.000 & 5.000 & 7.000 \\ 5.000 & 3.000 & 5.000 & 6.000 \\ 2.000 & 4.000 & 6.000 & 8.000 \end{pmatrix}$$

$$\widehat{B} = \left(\left(\widehat{b_{ij}}(\tau)\right)\right) = \begin{pmatrix} 1.000 & 3.000 & 5.000 & 7.000 \\ 2.000 & 4.000 & 6.000 & 8.000 \\ 3.000 & 4.000 & 5.000 & 6.000 \\ 7.000 & 8.000 & 9.000 & 10.000 \end{pmatrix}$$

We assume Terminal reward A = 100, Discounting Rate α = 0.5. The Expected reward is 0.163609 which is quite satisfactory. So $R(t,\alpha)$ is a satisfactory measure of expected reward for observed data. So, we can say that it is a good measure of occupational mobility.

Reward Structure Model without Discounting

In the previous section [3] we have considered a reward structure model for N categories of posts of faculty members. We have considered the same pattern of modelling here as in section [3]. In this model we have considered that individuals will enter in the *ith* post at time τ (calendar time) and they will enter post i and will be in post j with probability p_{ij}. They will stay at post i before getting a promotion to the next post j. But here we do not consider the situation where the payment of reward elongated over a long period that is we do not consider discounting in future reward. The holding time density function is $h_{ij}(\tau)$ at time τ. Individuals who are in post i and they have chosen the successor post j will earn a reward at the rate $y_{ij}(\sigma)$ at time σ (where σ is the calendar time, σ 6= τ). $y_{ij}(\sigma)$ is called the yield rate and it is the amount of money which is paid continuously for the occupancy of post i. $b_{ij}(\tau)$, is the bonus that individuals will get when they get promotion from post i to post j. Instead of considering the expected present value of the reward, here we have considered a total expected reward as we do not consider discounting in future reward. So, according to this model we have $v_i(t)$ to be the expected total reward that individuals will earn in the time t when they have started their job in the post i. So here we have $v_i(t) = v_i(t,0)$.

According to Howard[[20]], the expected total reward is

$$v_i(t,\alpha) = \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)d\tau \left[\int_0^t y_{ij}(\sigma)d\sigma + v_i(0)\right]$$
$$+ \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)d\tau \left[\int_0^\tau y_{ij}(\sigma)d\sigma + b_{ij}(\tau) + v_j(t-\tau)\right]$$
$$i = 1, 2. \dots N \quad t > 0 \tag{6}$$

Now $y_{ij}(\tau,0)$ is the reward that individuals will earn from the yield rate when they will spend time τ in post i before getting a promotion to the post j. Thus,

$$y_{ij}(\tau, 0) = \int_0^\tau y_{ij}(\sigma)d\sigma \tag{7}$$

Using equation[9] in equation[8] we get the expected total reward as [[20]]

$$v_i(t,\alpha) = \sum_{j=1}^{N} p_{ij} y_{ij}(t,0) \int_0^t h_{ij}(\tau)d\tau + v_i(0) \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)d\tau$$
$$+ \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau) \left[\int_0^\tau y_{ij}(\sigma)d\sigma + b_{ij}(\tau)\right] d\tau$$
$$+ \sum_{j=1}^{N} p_{ij} \int_{j=1}^{N} h_{ij}(\tau) v_j(t-\tau)d\tau \tag{8}$$

The expected total value of the future rewards at the time when individuals enter in the job is $v_j(t-\tau)$. It is impossible to calculate the total value of the future reward as the continuous change of Government policies. That is why we ignore the term $v_j(t-\tau)$. Now according to our consideration and assumption the expected total value of the reward is

$$R1_i(t,\alpha) = \sum_{j=1}^{N} p_{ij} y_{ij}(t,0) \int_0^t h_{ij}(\tau)d\tau + v_i(0) \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau)d\tau$$
$$+ \sum_{j=1}^{N} p_{ij} \int_0^t h_{ij}(\tau) \left[\int_0^\tau y_{ij}(\sigma)d\sigma - b_{ij}(\tau)\right] d\tau \tag{9}$$

The first part of the equation [9] represents the expected value of the total reward that is created by the yield rate when the first promotion occurs beyond time t. The last part of the above equation [9] represents the profit from yield and bonus which is associated with the promotion that is happened before time t.

$R1_i(t,\alpha)$ is considered as the occupational mobility measure without discounting. Many private concern as well as the government sectors where employees are recruited as a contractual basis follow this type of model. In those organisations employees are paid only due to their occupied post. They get a bonus according to as the organisation's rule. But they donot get any discounted reward though they have occupied a post for several years. Our proposed measure $R1_i(t,\alpha)$ can be used to represent the occupational situation as well as the payment criterion of those organisations.

CONCLUSIONS

This is the first part of this work we have attempted to find out the reward structure model due to the promotion of faculty members of Indian Universities. It will give us a clear idea about the promotion and corresponding reward gained by faculty members of Indian Universities. It will also give the idea of overall promotion and corresponding reward structure of foreign Universities or any other Institution. Here our measure can be considered as an occupational mobility measure based on the reward structure model when discounting is considered. We fit our measure for real life situation. It is well fitted.

In the last part of this work we have also found out the reward structure model for the organisations where discounting in future reward cannot be considered. In those organisations employees get a reward due to yield and bonus rate. Government sectors where employees are recruited on a contractual basis also follow this type of reward structure model. We proposed the measure of occupational mobility based on reward structure where discounting in future reward do not consider. We cannot fit this measure for the lack of data. But we make sure that our measure is well fitted for those observed data. In our future work we will definitely try to fit our second measure for real life data. But still we can make sure that our proposed measure will be well fitted of those organisations where discounting in future reward is not considered. Our proposed measure can give the true payment situation due to the promotion of employees in those situations.

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